

SECTION C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. State Cayley – Hamilton theorem and show that it

is satisfied by the matrix  $A = \begin{bmatrix} -11 & -10 & 6 \\ 5 & 4 & -5 \\ -20 & -20 & 4 \end{bmatrix}$ .

17. What do you mean by symmetric and antisymmetric tensors? Show that any contravariant (or) covariant tensor of second rank can be expressed as the sum of a symmetric and an antisymmetric tensor of the same rank.

18. Solve the differential equation.

$$\frac{d^2 y}{dx^2} + 4 \frac{dy}{dx} + 3y = 65 \cos 2x.$$

19. Prove that :  $e^{\frac{x}{2} \left[ \frac{1}{z} \right]} = \sum_{-\infty}^{+\infty} z^n J_n(x).$

20. Derive Green's function for three dimensional Helmholtz equation.

APRIL/MAY 2024

GPH11/DPH11 — MATHEMATICAL  
PHYSICS — I

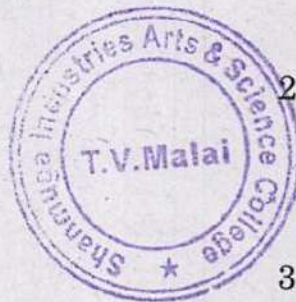
Time : Three hours

Maximum : 75 marks

SECTION A — (10 × 2 = 20 marks)

Answer ALL questions.

1. Show that the vectors (1, 2, -3), (1, 3, -2) and (2, -1, 5) are linearly independent.
2. What is the characteristic equation of a matrix  $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ ?
3. What do you mean by real and Dummy indices in tensor analysis?
4. If  $A_{ij}$  is an antisymmetric tensor, what is the value of  $A_{11}$ ? How?
5. Solve :  $(1 - x^2)(1 - y)dx = xy(1 + y)dy.$





6. Solve the differential equation

$$\frac{d^2 y}{dx^2} - 3 \frac{dy}{dx} + 2y = 0.$$

7. Prove that :  $P_{2n+1}(0) = 0$ .

8. Give the Rodrigue's formula for Hermite polynomials.

9. State two properties of Delta function.

10. What is Green's function? Give its symmetry properties.

SECTION B — (5 × 5 = 25 marks)

Answer ALL questions.

11. (a) Show that the vectors  $(u+v)$ ,  $(v-v)$  and  $(u-2v+w)$  are linearly independent provided  $(u, v, w)$  are linearly independent.

Or

- (b) Find the eigen values and normalized eigen

vectors of the matrix  $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$ .

12. (a) Prove that the addition of tensors is commutative and associative.

Or

- (b) Mention the properties of Levi - Civita symbol, and show that  $\sum_{jk=1}^3 \epsilon_{ijk} \epsilon_{ljk} = 2\delta_{il}$ .

13. (a) Solve the differential equation.

$$\frac{d^2 y}{dx^2} - y = \frac{2}{1+e^x}.$$

Or

- (b) Solve the equation

$$\frac{dy}{dx} + xy = x^3 y^3.$$

14. (a) Show that  $P_n(x)$  is the coefficient of  $z^n$  in the expansion of  $[1 - 2xz + z^2]^{-1/2}$  in ascending powers of  $z$ .

Or

- (b) Obtain the generating function for Hermite polynomials.

15. (a) Show that  $f(a, x) = \frac{1}{a} f(x)$ .

Or

- (b) Find one dimensional Green's function for the boundary value problem.

$$y''(x) + y(x) = f(x), y(0) = 0 \text{ and } y(1) = 0.$$

